Ch. 6 Photons, Electrons, and Atoms

References:

1. Young & Freedman, “University Physics”, 13th ed. Ch. 38, 39

2. Halliday et al., “Principles of Physics”, 9th ed. Ch. 38, 39

Outline

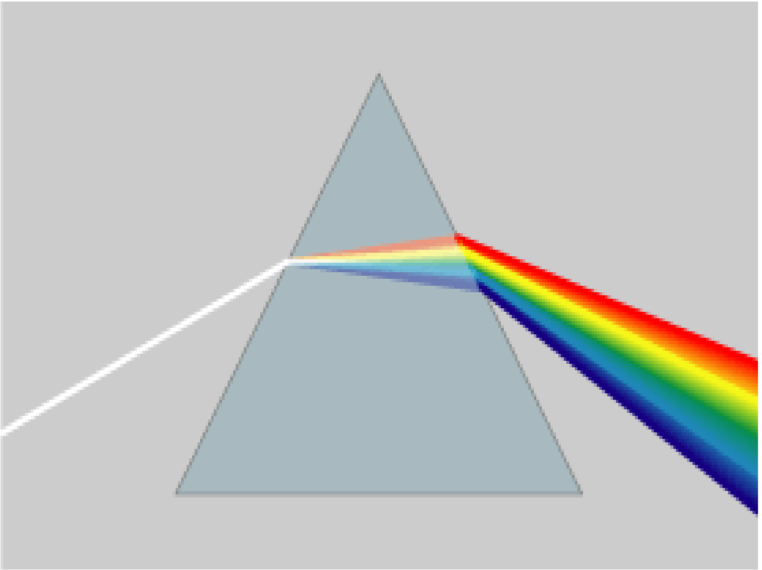
6.1 The Limits of Classical Physics
6.2 The Photoelectric Effect
6.3 The Nuclear Atom
6.4 The Bohr Model
6.5 The Compton Effect

6.1 The Limits of Classical Physics

Great challenges facing physicists around 1900:

- ***Line Spectra***

If the light source is a hot solid, the spectrum is continuous.



If the light source is a heated gas, we will have a line spectrum.

- ***The Hydrogen Spectrum***

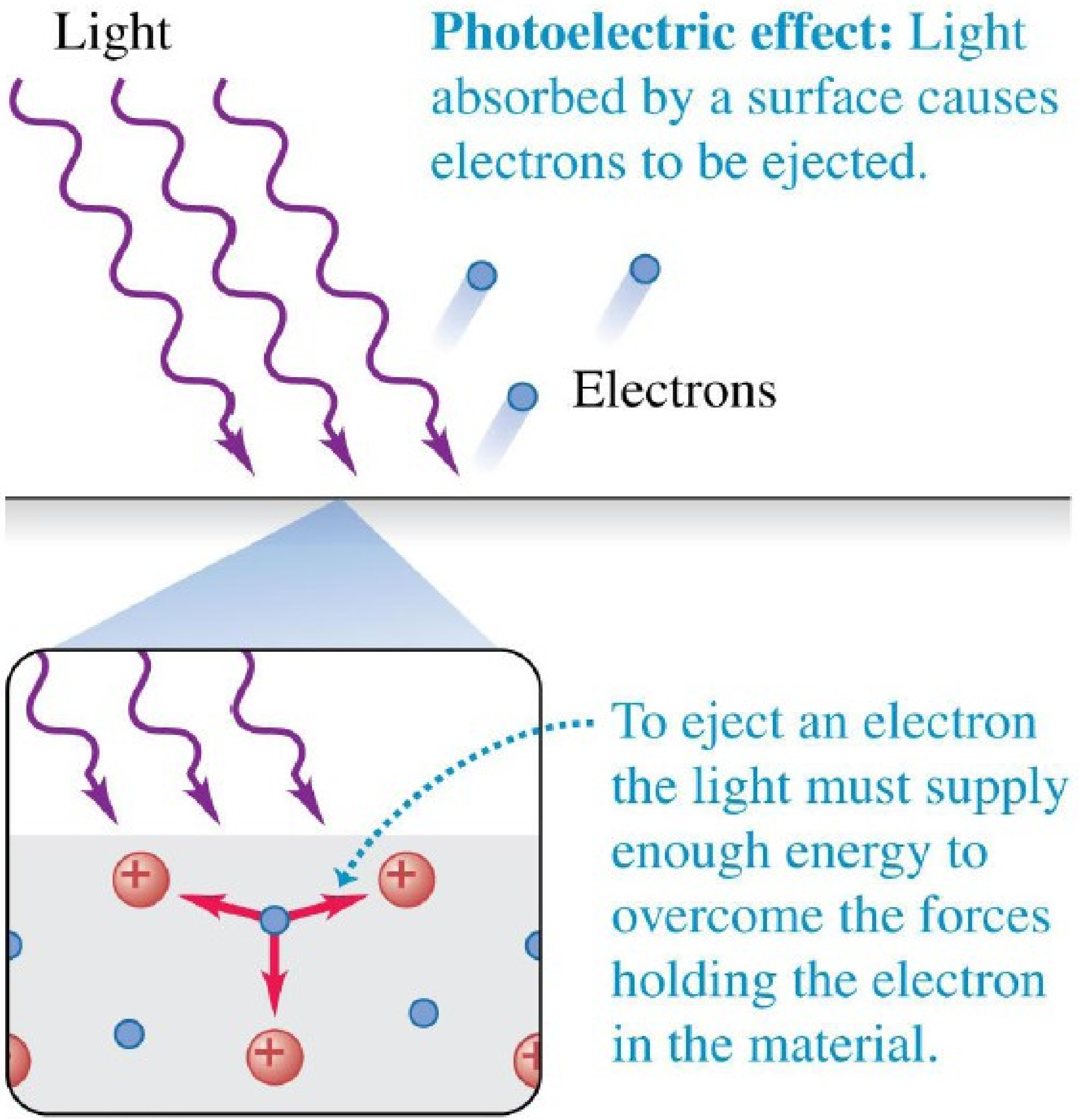
In 1885, J. Balmer found a formula that gives the wavelengths of the lines (Balmer series):

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad (n = 3, 4, 5, \ldots)$$

$R = 1.097 \times 10^7\ \mathrm{m}^{-1}$ (Rydberg constant)

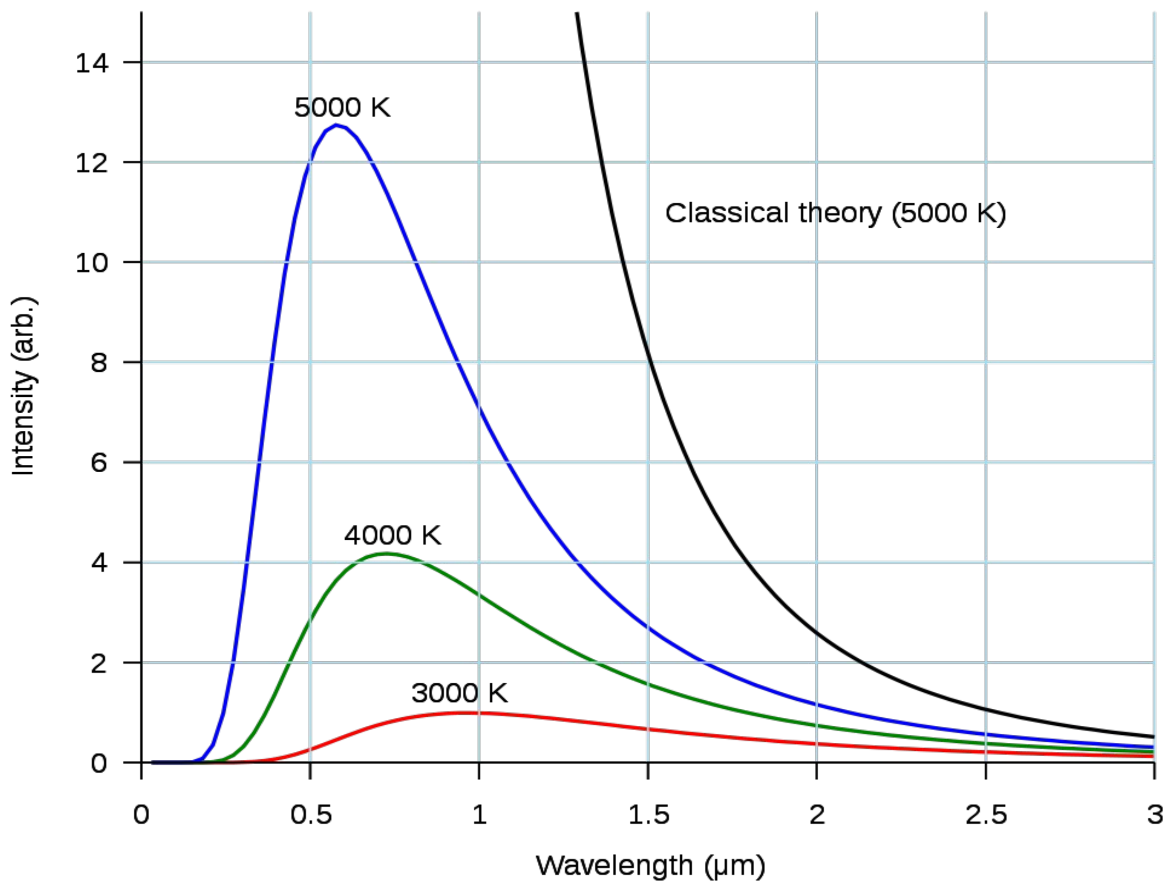
Can this formula be explained?

- ***Photoelectric Effect***



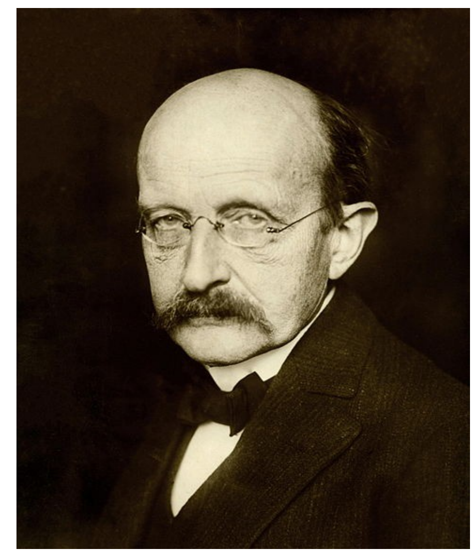
• ***Black Body Radiation*** (Optional)

The spectral distribution of radiation from a black body:



Planck radiation law (1900)

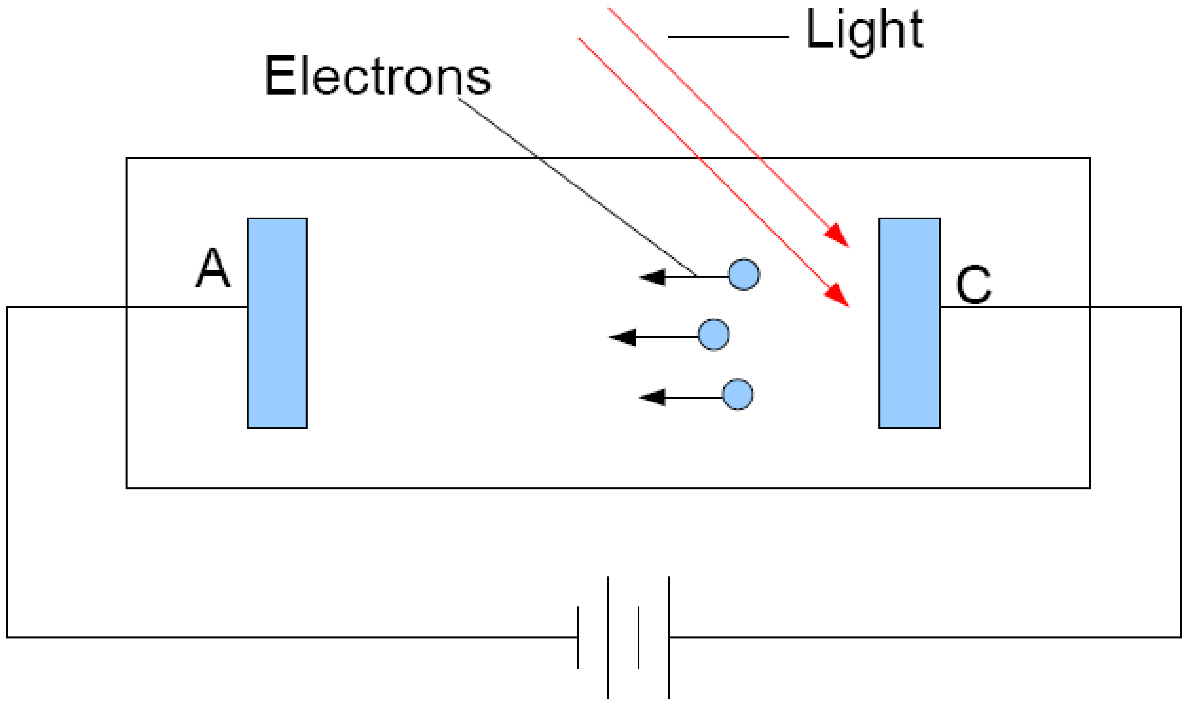
$$I(\lambda)=\frac{2\pi h c^2}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)}$$



Max Planck
(1858-1947)

(More in thermal physics
and statistical mechanics
courses)

6.2 The Photoelectric Effect



When the voltage is increased to a certain value, no more electrons are emitted and the current drops to zero.

Experimental findings:

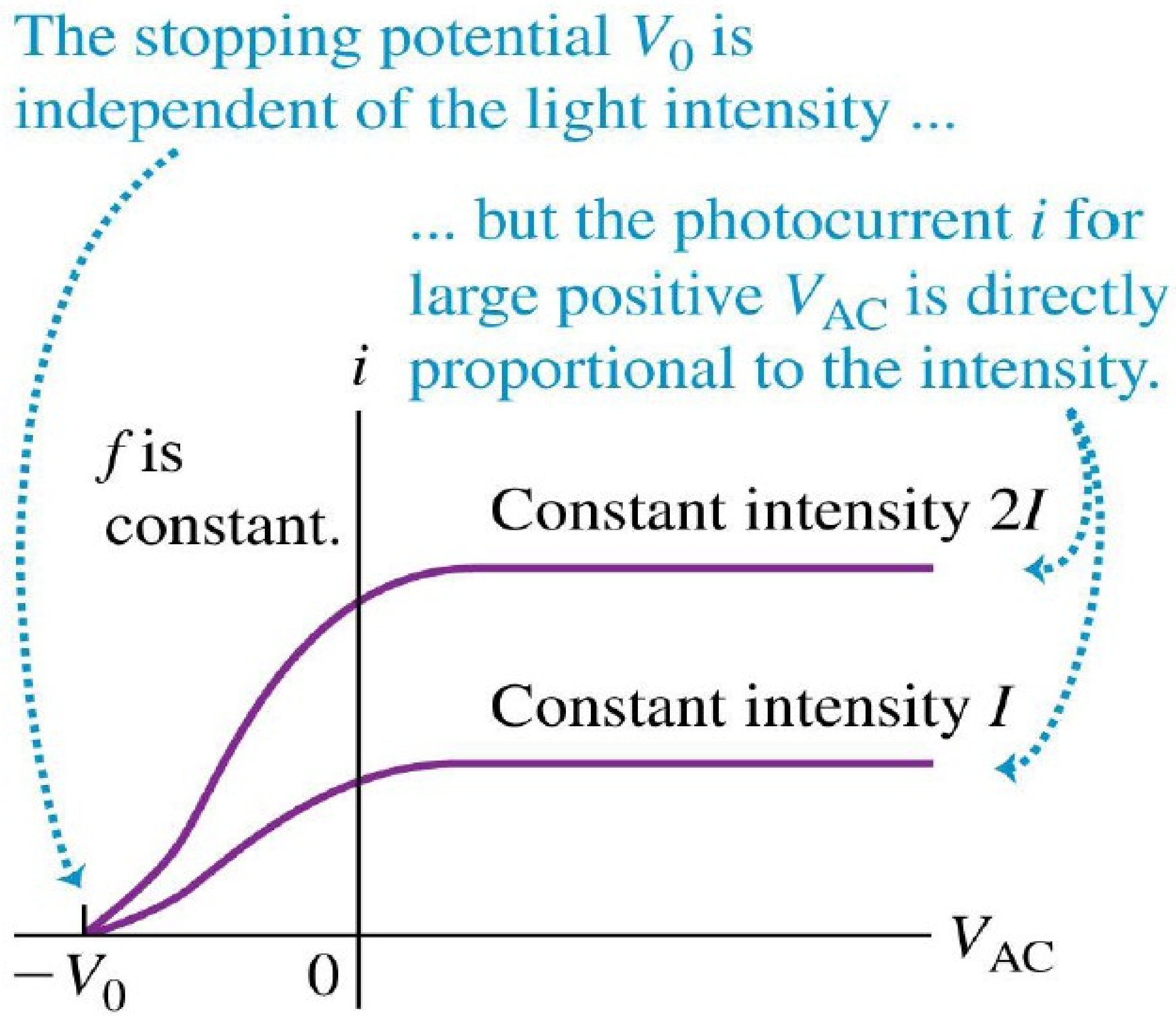
1. There is a minimum frequency f_{min} of the light, below which no electrons are emitted.
   (f_{min} depends on the material)

2. When $f > f_{\text{min}}$, electrons are emitted.
   There exists a stopping potential V_0 such that, if $V_{AC} = -V_0$, the current stops.

   => Max. KE of emitted electrons:

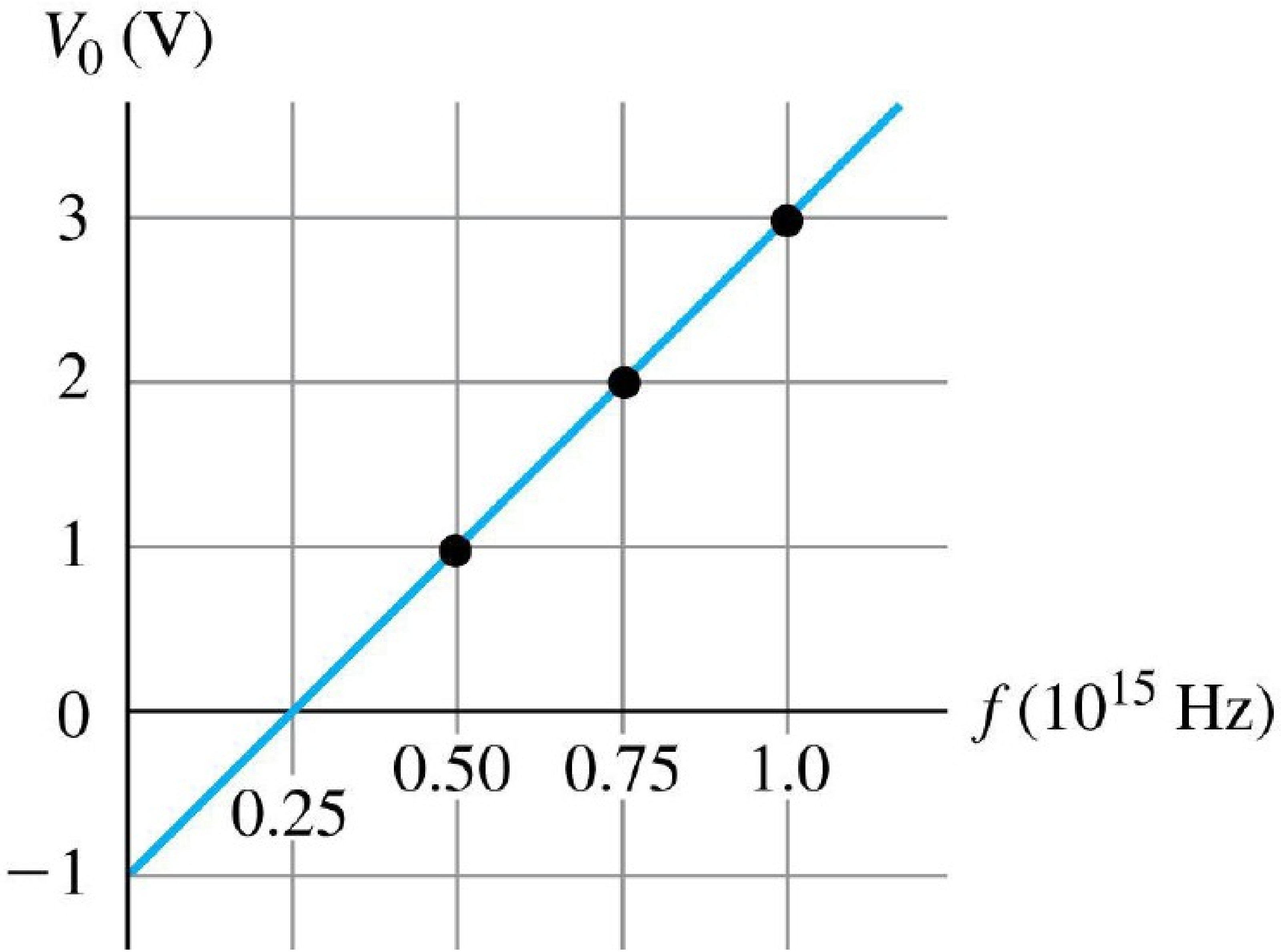
$$K_{\max} = \frac{1}{2} m v_{\max}^2 = e V_0$$

3. For a fixed frequency, V_0 is independent of the intensity of light.



Classical physics => When intensity increases, electrons should gain more energy, and hence increasing V_0.

4. For a fixed intensity, V_0 depends linearly on frequency.



- ***Einstein's Photon Explanation***

Einstein (1905) postulated that a beam of light consists of small packages of energy called photons.

Energy of a photon:

$$E = h f = \frac{h c}{\lambda}$$

where the Planck's constant

$$h \approx 6.626 \times 10^{-34} \text{ J s}$$

Let ϕ = minimum energy needed to remove an electron from the surface (Work function)

Max KE of electron: $K_{max} = \frac{1}{2} m v_{max}^2 = h f - \phi$

Recall: $K_{max} = e V_0$

$$e V_0 = h f - \phi$$

Note: A graph of V_0 vs f => h/e and ϕ

Example:

Element	Work Function (eV)
Aluminum	4.3
Carbon	5.0
Copper	4.7
Gold	5.1
Nickel	5.1
Silicon	4.8
Silver	4.3
Sodium	2.7

Note: Electron volt $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

=> $h = 6.626 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-15} \text{ eV s}$

- ***Photon Momentum***

Recall: $E^2 = (mc^2)^2 + (pc)^2$

For photons, m = 0

=> $E = pc$

Einstein's postulate: $E = hf = \frac{hc}{\lambda}$

=> $$p = \frac{hf}{c} = \frac{h}{\lambda}$$

6.3 The Nuclear Atom

- ***Situation in* 1910**

1. J. J. Thomson had discovered the electron and measured the charge-to-mass ratio (e/m) in 1897.

2. Millikan had completed his first measurement of the electron charge in1909. (Millikan oil drop experiment)

3. Almost all of the mass of an atom had to be associated with positive charge, not with the electrons.

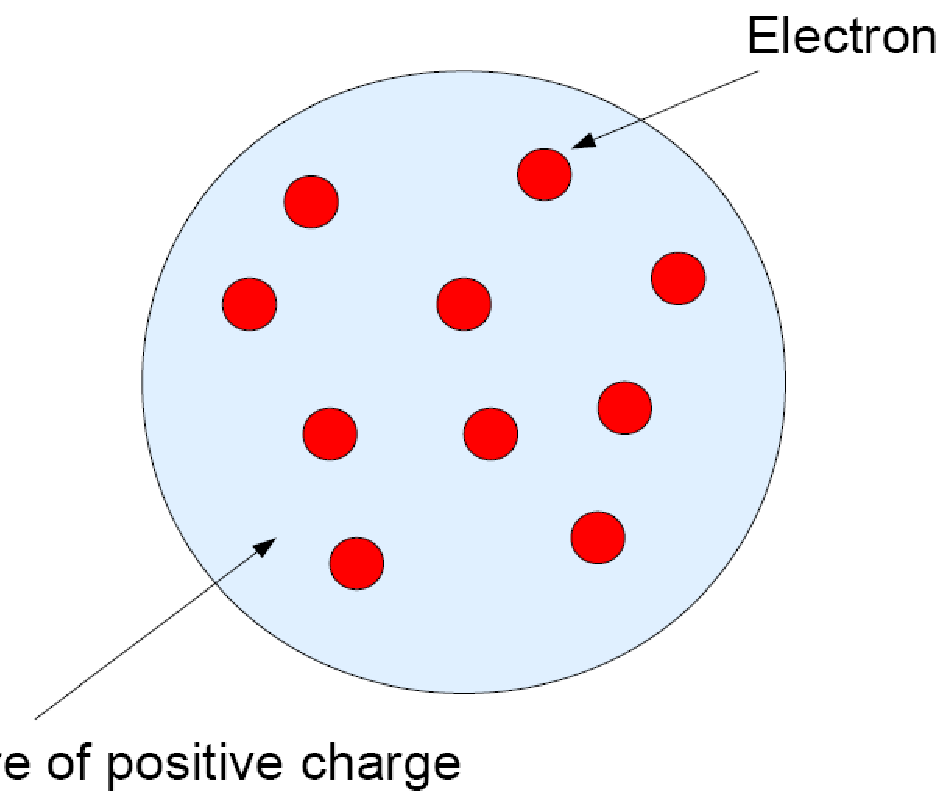
4. Overall size of atoms is ~O(10^{-10} m).

5. All atoms except hydrogen contain more than one electrons.

What was not known then was how the mass and charge were distributed within the atom.

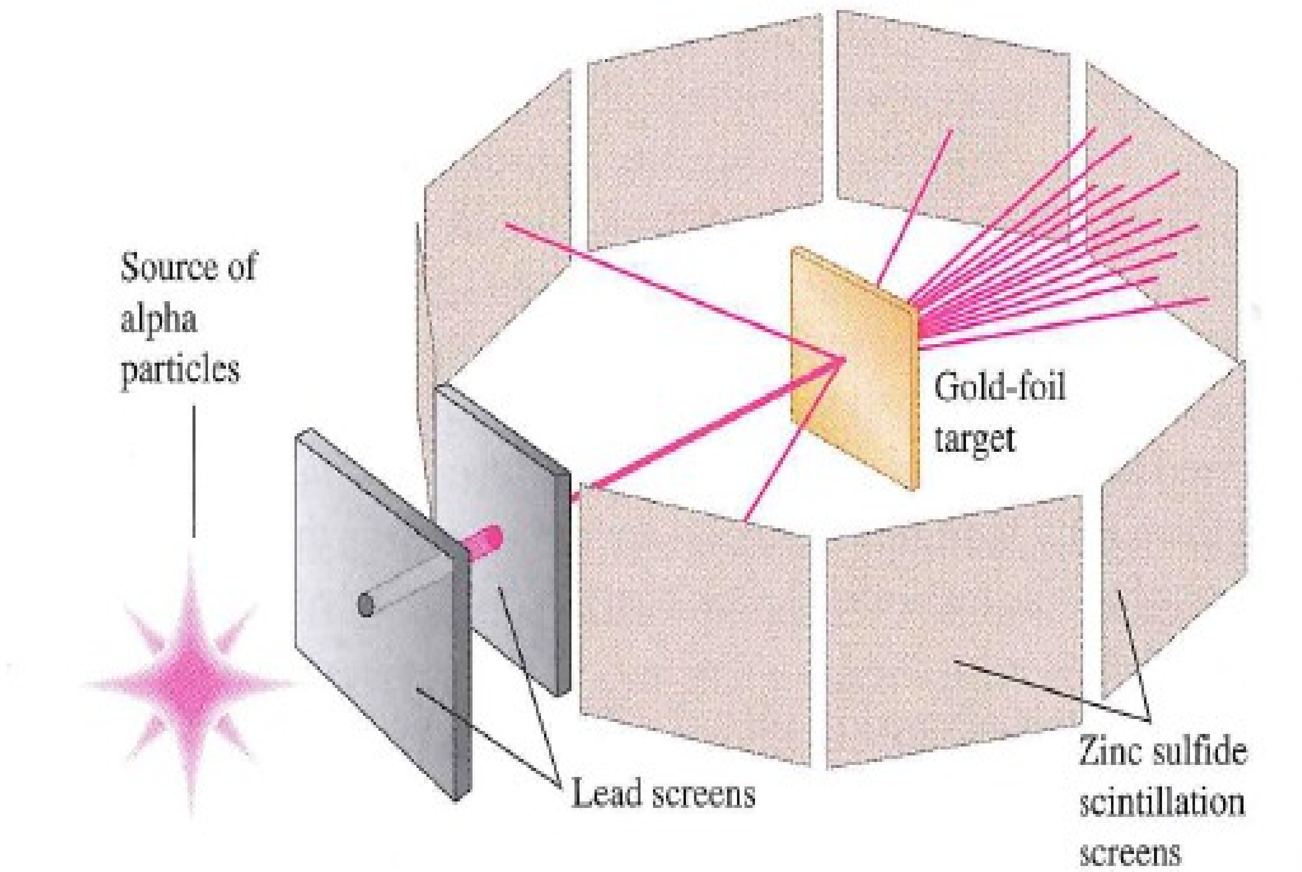
Thomson model of the atom (1904):
Electrons embedded in a cloud of positive charged matter.

J. J. Thomson
(1856-1940)



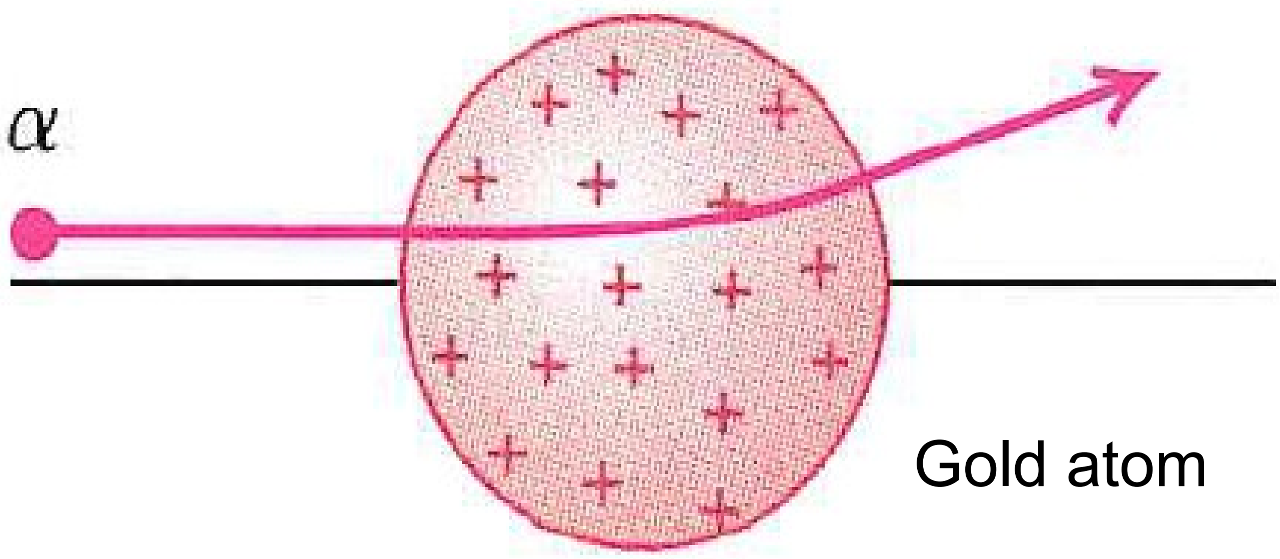
• ***Rutherford Scattering Experiments***

The first experiments designed to probe the interior structure of the atom were carried out by E. Rutherford and his students H. Geiger and E. Marsden (1909)



Rutherford
(1871-1937)

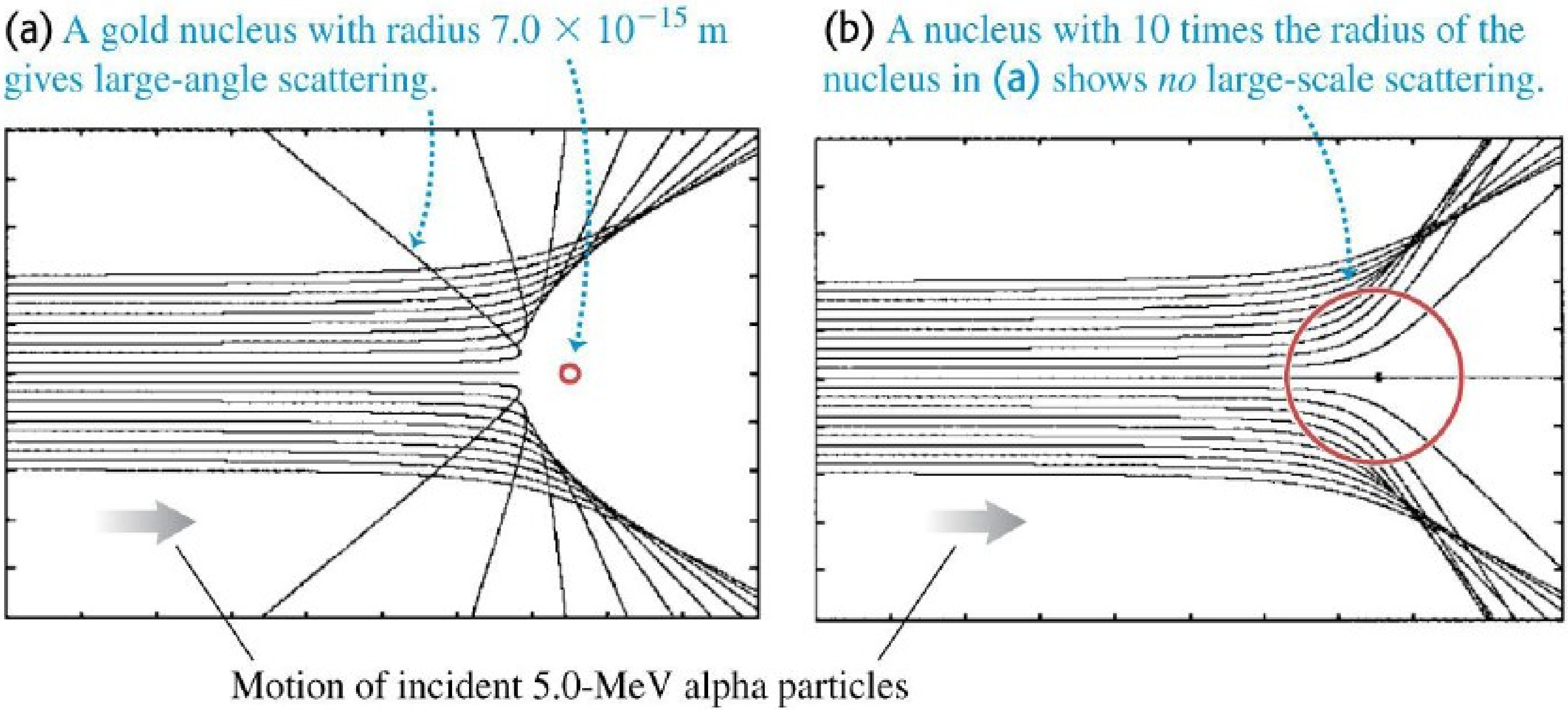
In the Thomson model, the alpha particle is expected to be scattered through only a small angle.



Experimental results:

Some alpha particles were scattered by nearly 180°.

Computer simulation of scattering of 5 MeV alpha particles from a gold nucleus:

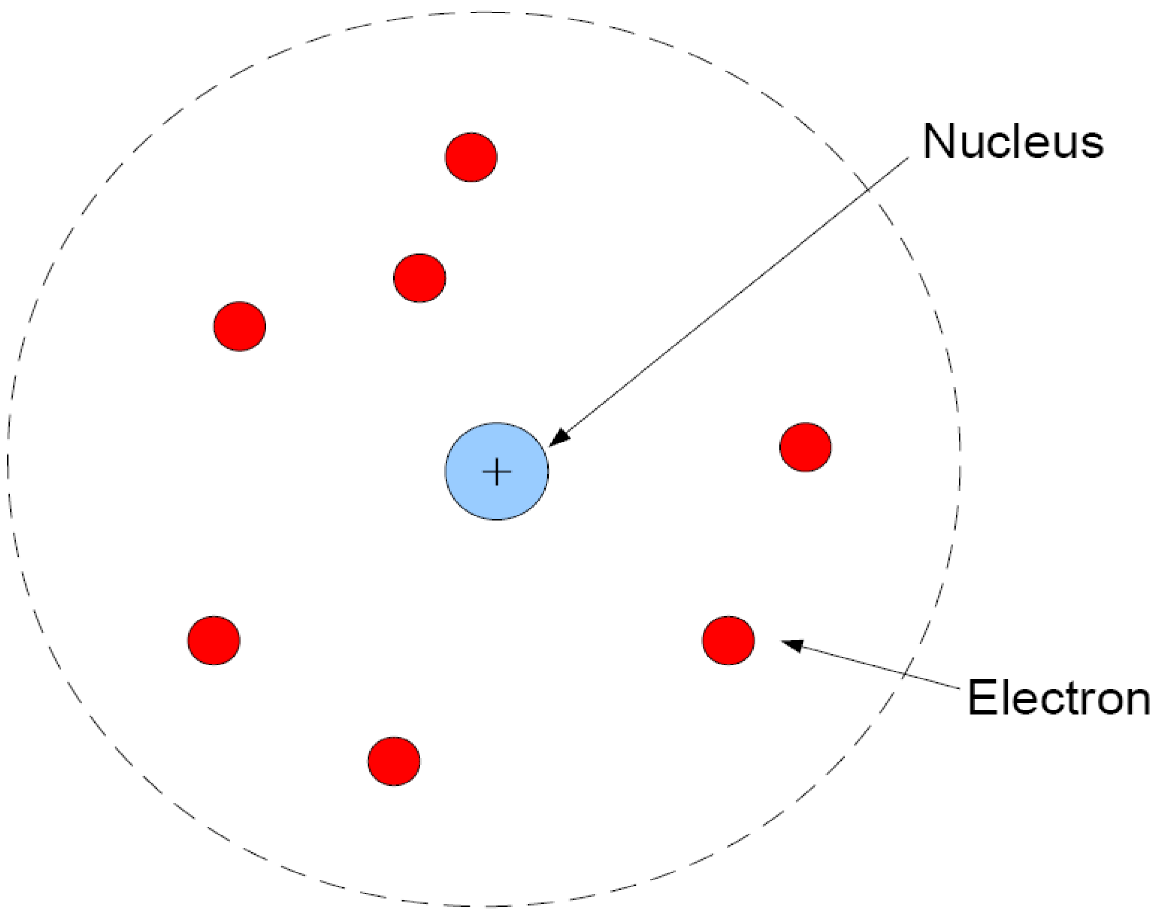


Rutherford later wrote :
"It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

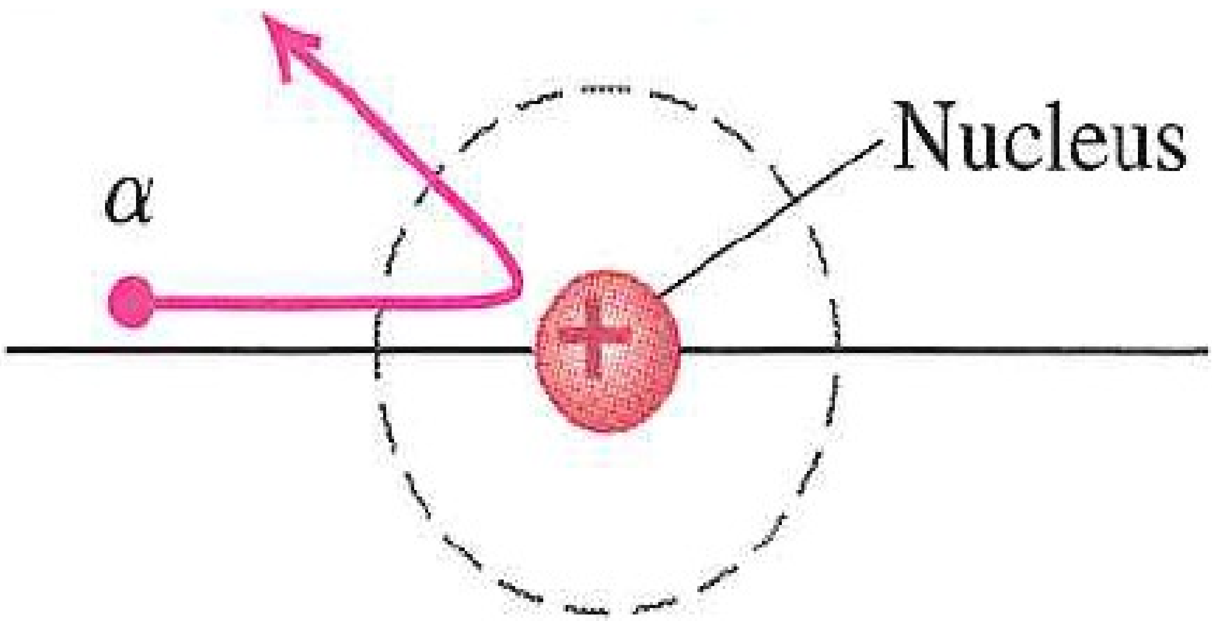
Rutherford model of the atom (1911):

The positively charged nucleus contains almost all of the total mass of the atom (~99.95%).

Its diameter is less than 10^{-14} m
(only about 10^{-12} of the total volume of the atom).



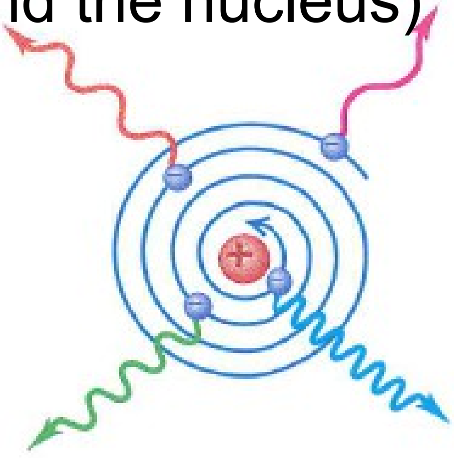
In Rutherford model, an alpha particle can be scattered through a large angle by the compact, positively charged nucleus:



6.4 The Bohr Model

• ***Problems with the Rutherford model***

1. What kept the negatively charged electrons at large distances from the positively charged nucleus?
(Rutherford suggested that electrons revolve around the nucleus)



2. Classical EM theory
=> accelerating charges emit EM radiation.
=> The energy of an orbiting electron should decrease, and its orbit should become smaller and smaller.

3. Frequency of the EM radiation should equal the frequency of revolution. The spectrum should be continuous.

- ***Stable Electron Orbits***

In 1913, Bohr postulated that an electron in an atom can move around the nucleus only in certain circular stable orbits, without emitting radiation. These allowed orbits are called stationary orbits.

Bohr also argued that the angular momentum of the electron is given by

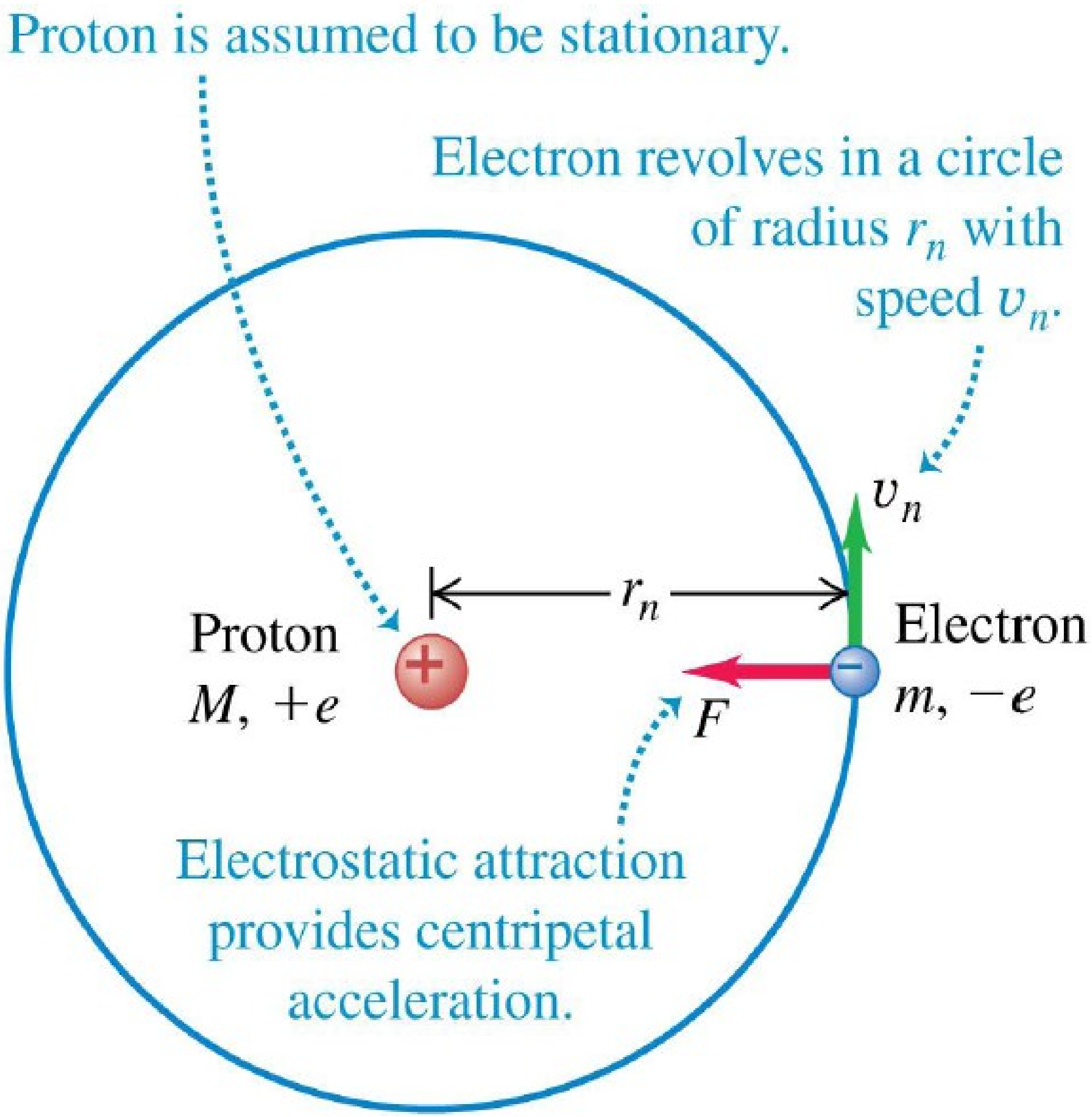
$$L = m v_n r_n = n \frac{h}{2\pi}$$

Quantization of angular momentum

where n = 1, 2, 3,... (principle quantum number)

Note: It is common to define $\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34}\ \mathrm{J\ s}$

Niels Bohr
(1885-1962)



Hydrogen atom with quantization assumption:

Coulomb force on the electron $$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n^2}$$

Newton's second law => $$\frac{m v_n^2}{r_n} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n^2}$$

Quantization of angular momentum =>

$$r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2}$$ Orbit radii

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2 n h}$$ Orbital speed

The smallest orbit radius (n = 1):

$$a_0 = \frac{\epsilon_0 h^2}{\pi m e^2} = 5.29 \times 10^{-11}\ \mathrm{m}$$ Bohr radius

- ***Hydrogen Energy Levels in the Bohr Model***

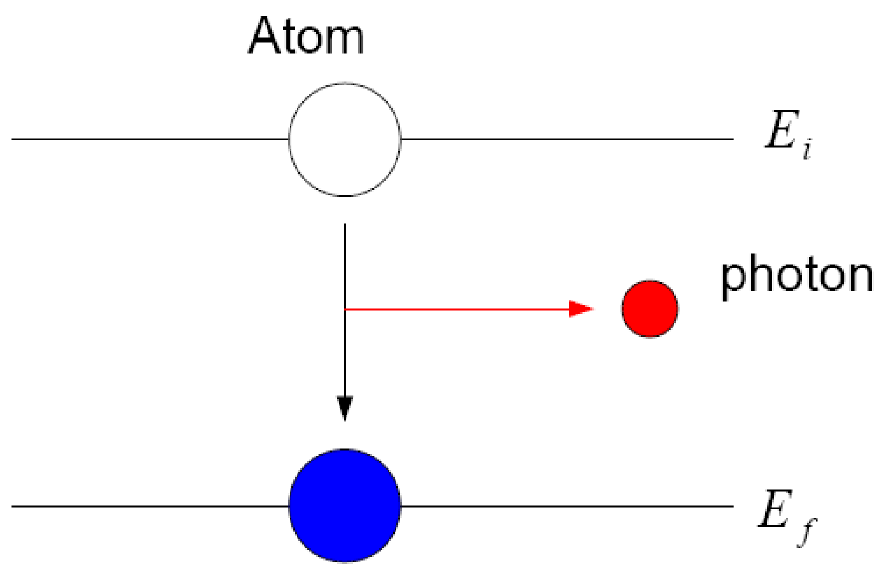
Kinetic energy: $$K_n = \frac{1}{2} m v_n^2 = \frac{1}{\epsilon_0^2} \frac{m e^4}{8 n^2 h^2}$$

Potential energy: $$U_n = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_n} = -\frac{1}{\epsilon_0^2} \frac{m e^4}{4 n^2 h^2}$$

Total energy: $$E_n = K_n + U_n$$

$$E_n = -\frac{1}{\epsilon_0^2} \frac{me^4}{8 n^2 h^2} = -\frac{13.6 \text{ eV}}{n^2} \qquad (n = 1, 2, 3, \ldots.)$$

- ***Photon Emission***



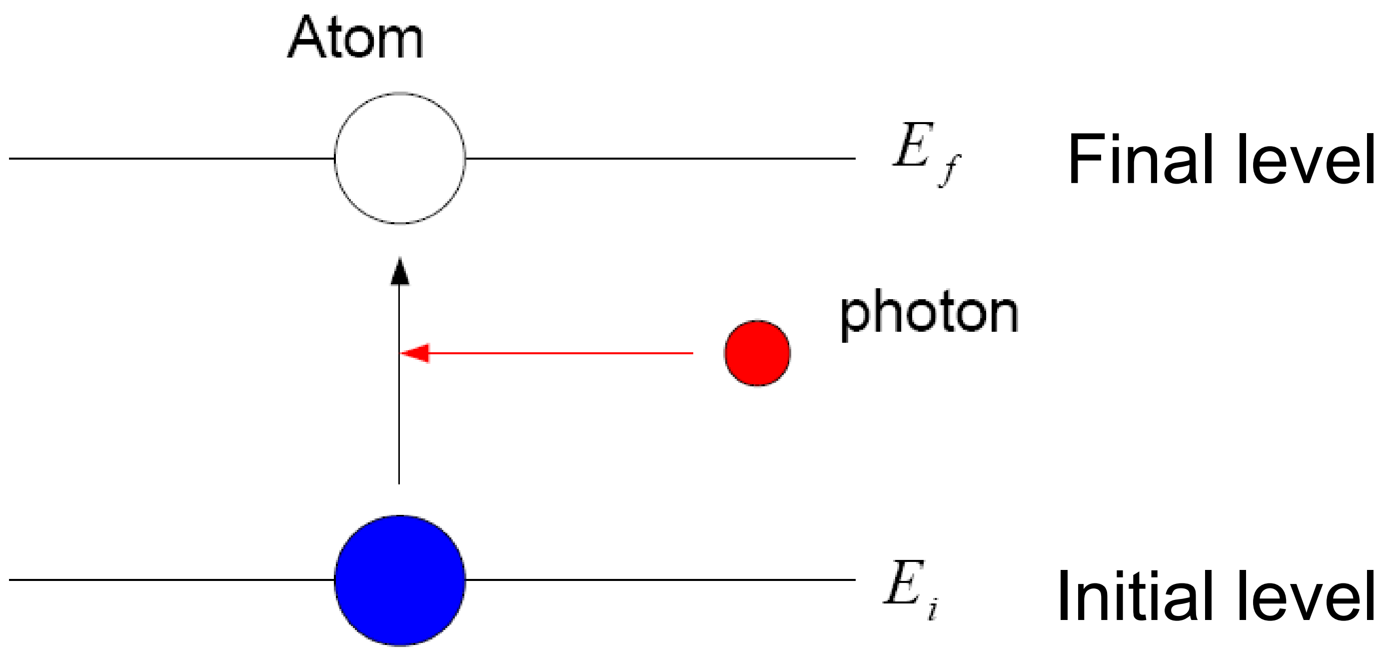
Bohr's hypothesis:

An atom can make a transition from one level to a lower level by emitting a photon with energy

$$hf = \frac{hc}{\lambda} = E_i - E_f$$

- ***Photon Absorption***

A photon is absorbed when an atom makes a transition from a lower energy level to a higher level.



Energy of the absorbed photon:

$$hf = \frac{hc}{\lambda} = E_f - E_i$$

- ***The Hydrogen Spectrum***

Consider a downward transition: $E_n \rightarrow E_m \quad (n > m)$

Energy of the emitted photon: $E_{\text{photon}} = \dfrac{hc}{\lambda} = E_n - E_m$

$$\Rightarrow \qquad \frac{1}{\lambda} = \frac{13.6\ \text{eV}}{hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

Recall: Balmer series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right) \qquad (n = 3, 4, 5, \ldots)$

$$\Rightarrow \quad m = 2 \quad , \quad R = \frac{13.6\ \text{eV}}{hc} = 1.097 \times 10^7\ \text{m}^{-1}$$

Summary:

1. Bohr model predicts that the energy of an electron in a H atom is quantized and given by

$$E_n = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2} = -\frac{13.6\ \text{eV}}{n^2}$$

2. n = 1 is the ground state:

$$E_1 = -13.6\,\text{eV}$$

$$r_1 = a_0 = 5.29\times10^{-11}\,\text{m} \quad \text{(Bohr radius)}$$

3. To remove the electron completely (transition from n=1 to n=∞), the energy required is

$$\Delta E = E_\infty - E_1 = 13.6\ \text{eV} \quad \text{(Ionization energy)}$$

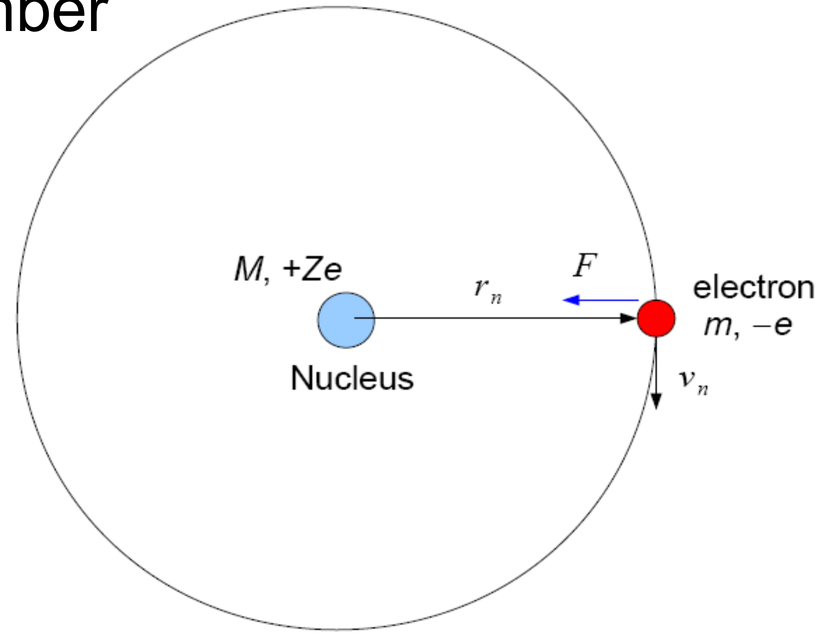
• *Hydrogen-Like 'Atoms'*

We can extend the Bohr model to other one-electron 'atoms' (e.g., singly ionized He^+, doubly ionized Li^{2+},..etc)

Coulomb force:

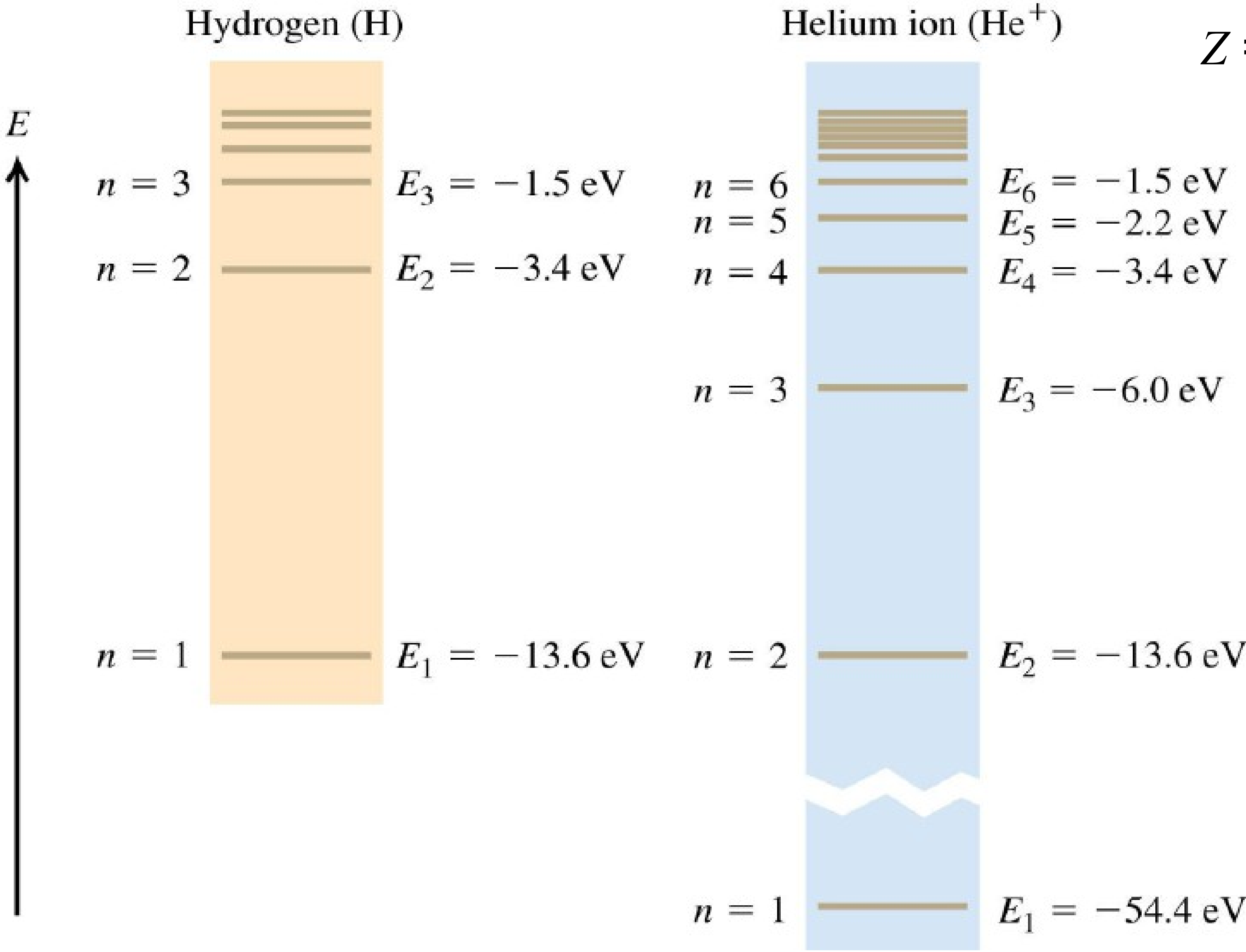
Atomic number

$$F = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_n^2}$$



The effect is to replace e^2 by Ze^2 in the previous analysis:

$$E_n = -\frac{1}{\epsilon_0^2}\frac{mZ^2e^4}{8n^2h^2} = -Z^2\frac{13.6\ \text{eV}}{n^2}$$


Hydrogen (H)
Helium ion (He^+)
E
n = 3
$E_3 = -1.5$ eV
n = 2
$E_2 = -3.4$ eV
n = 1
$E_1 = -13.6$ eV
n = 6
$E_6 = -1.5$ eV
n = 5
$E_5 = -2.2$ eV
n = 4
$E_4 = -3.4$ eV
n = 3
$E_3 = -6.0$ eV
n = 2
$E_2 = -13.6$ eV
n = 1
$E_1 = -54.4$ eV

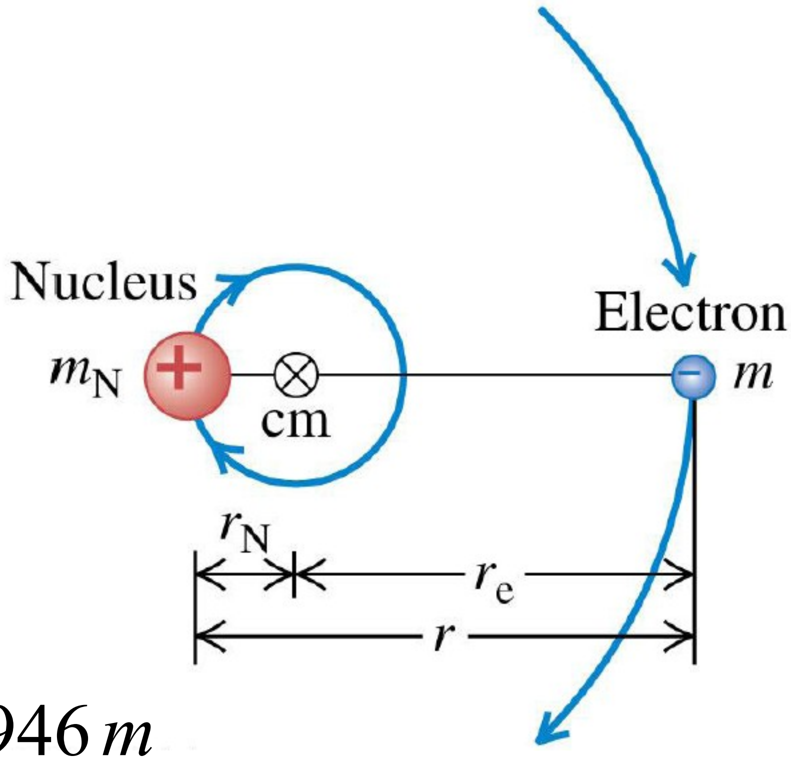
$Z = 2$

• *Nuclear Motion and the Reduced Mass of an Atom*

In general, the electron and the nucleus both orbit about their common center of mass. This effect can be taken into account by using the reduced mass of the system:

$$m_r = \frac{M\,m}{M+m}$$

=> $$E_n = -\frac{1}{\epsilon_0^2}\frac{m_r Z^2 e^4}{8 n^2 h^2}$$



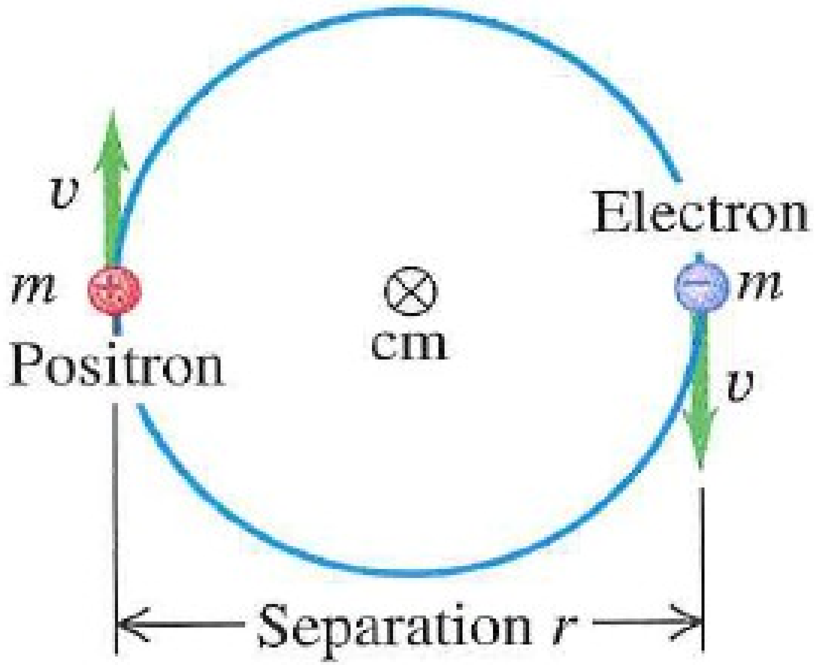
For hydrogen atom:

$$m_r = \frac{m_p m}{m_p + m} = \frac{m(1836.2\,m)}{m + 1836.2\,m} = 0.99946\,m$$

(small correction)

Example: Positronium 'atom'

Positron is a anti-particle of electron
(mass = m ; charge $+e$)



$$m_r = \frac{m^2}{m+m} = \frac{m}{2}$$

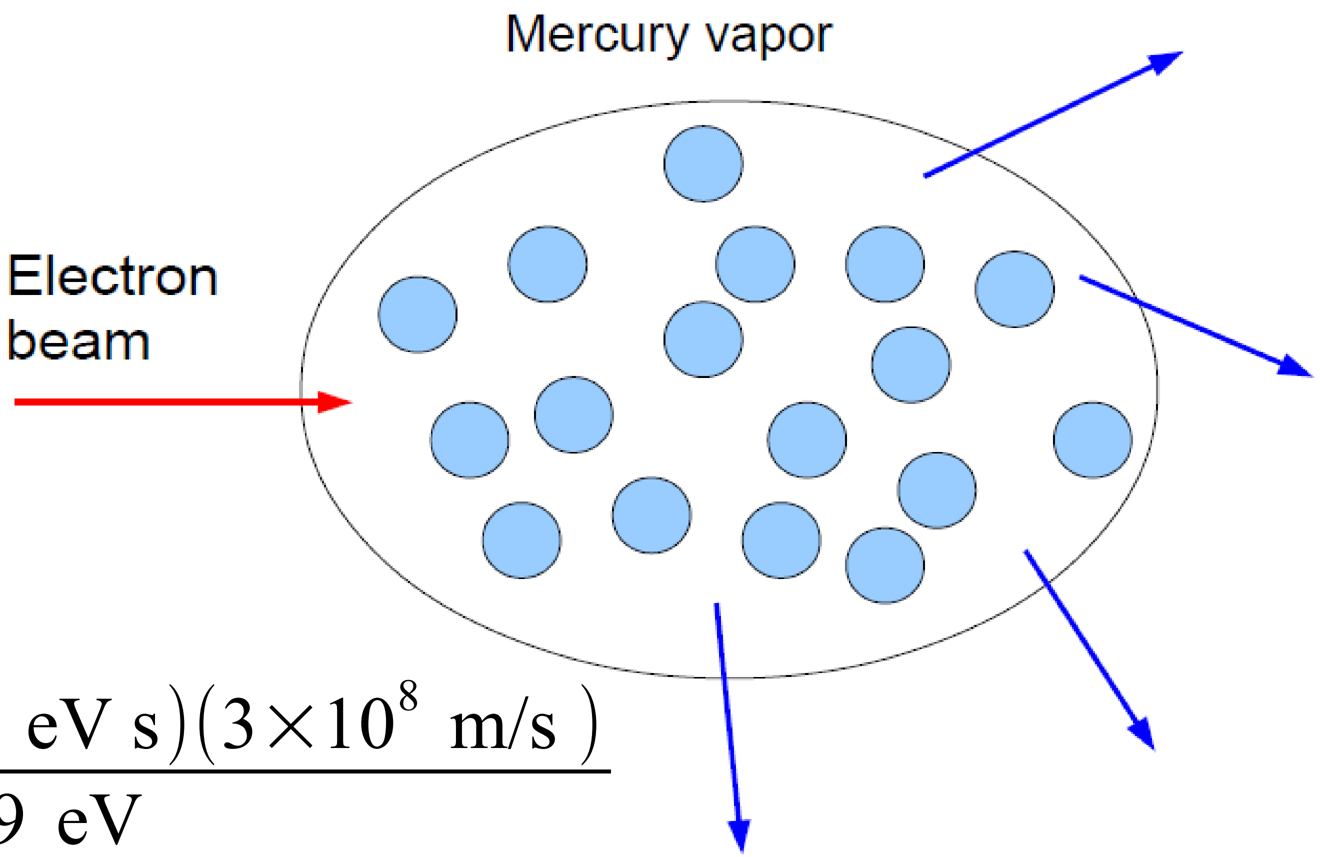
$$E_n = -\frac{1}{\epsilon_0^2} \frac{m_r Z^2 e^4}{8 n^2 h^2} = -\frac{13.6\ \mathrm{eV}}{2 n^2}$$

The existence of positronium atom was confirmed by observation of the corresponding spectrum lines.

- ***The Franck-Hertz Experiment*** **(1914):**

Are energy levels real?

When electrons with KE = 4.9 eV passed through mercury vapor, the vapor emitted UV light of wavelength 250 nm

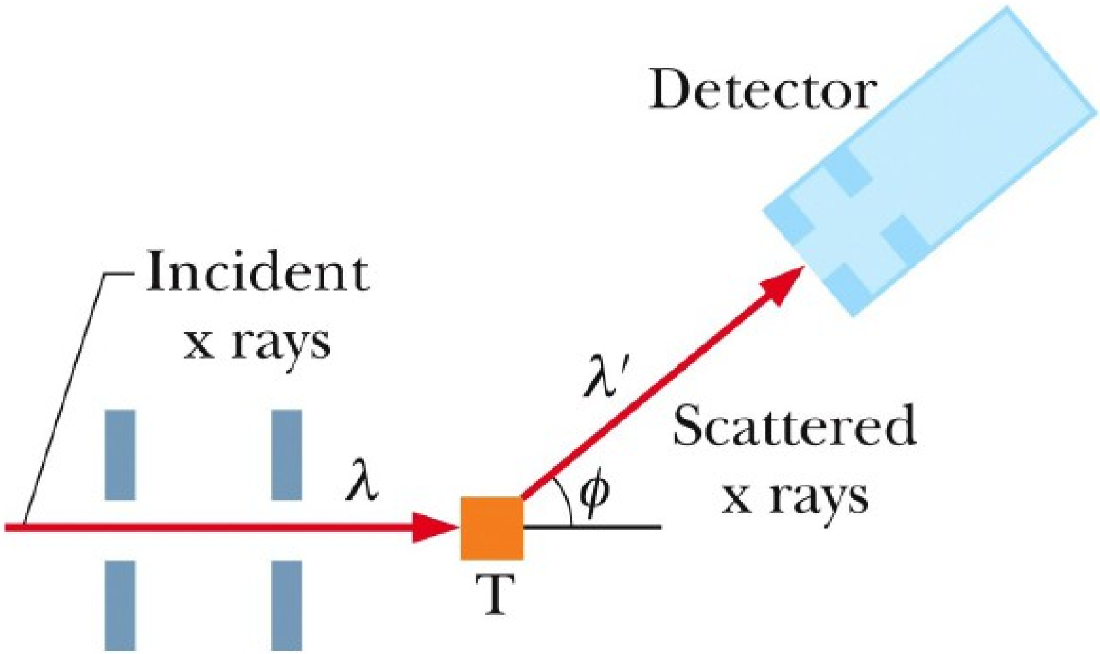


$$\lambda = \frac{hc}{E}$$

$$= \frac{\left(4.136\times10^{-15}\ \text{eV s}\right)\left(3\times10^{8}\ \text{m/s}\right)}{4.9\ \text{eV}}$$

$$= 250\ \text{nm}$$

6.5 The Compton Effect



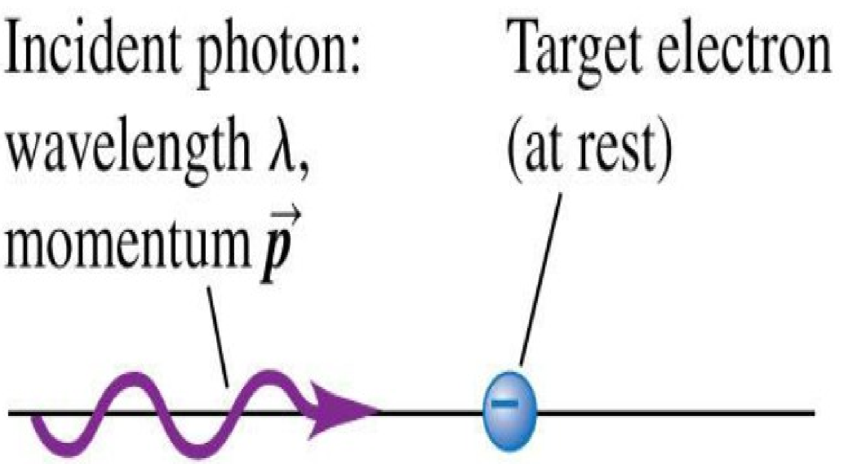
Classical EM theory => the scattered wave has the same wavelength as the incident wave

Compton's finding: some of the scattered wave has longer wavelength than the incident wave

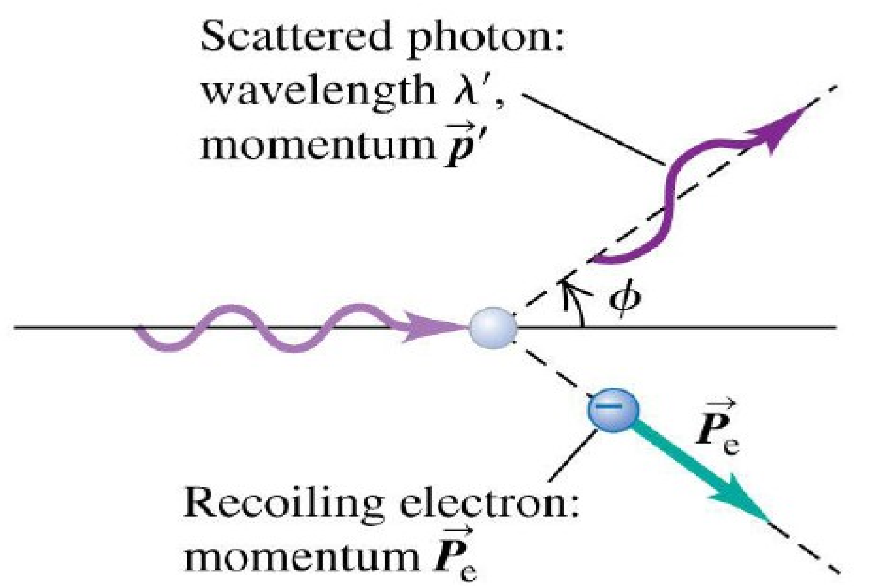
(1923)

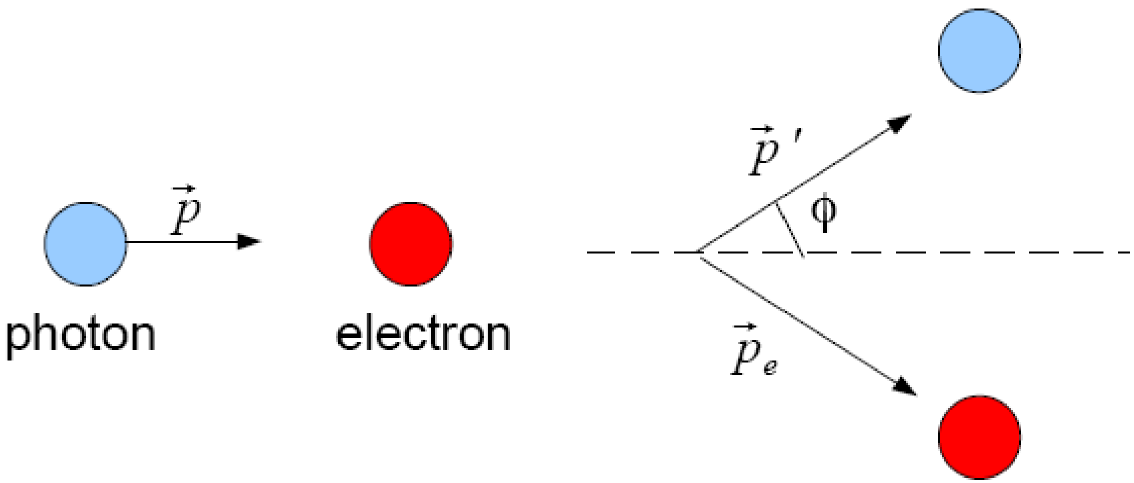
Consider the scattering process as a collision between the incident photon and a target electron:

(a) Before collision: The target electron is at rest.



(b) After collision: The angle between the directions of the scattered photon and the incident photon is ϕ.





m = electron's rest mass

Energy conservation: $pc + mc^2 = p'c + E_e$ (Eq. 1)

Note: $E_e^2 = (mc^2)^2 + (p_e c)^2$ (Eq. 2)

Momentum conservation: $\vec{p} = \vec{p}\,' + \vec{p}_e$

$$\Rightarrow \quad p_e^2 = (\vec{p} - \vec{p}\,') \cdot (\vec{p} - \vec{p}\,')$$
$$= p^2 + p'^2 - 2\,\vec{p} \cdot \vec{p}\,'$$
$$= p^2 + p'^2 - 2\,p\,p'\cos\phi \qquad \text{(Eq. 3)}$$

(Eq. 1)-(Eq. 3) =>

$$(pc - p'c + mc^2)^2 - m^2c^4 = (pc)^2 + (p'c)^2 - 2pp'c^2\cos\phi$$

$$2mc^3(p - p') = 2pp'c^2(1 - \cos\phi)$$

$$\frac{1}{p'} - \frac{1}{p} = \frac{1}{mc}(1 - \cos\phi)$$

For photon: $E = pc = hc/\lambda$

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

Compton scattering

For large m (eg, scattering by the whole atom): $\lambda' \approx \lambda$

Intensity as a function of wavelength for photons scattered at different angles: